

OPTIONS, FUTURES, AND OTHER DERIVATIVES

John C. Hull

*Maple Financial Group Professor of Derivatives and Risk Management
Joseph L. Rotman School of Management
University of Toronto*

ELEVENTH EDITION

GLOBAL EDITION



Harlow, England • London • New York • Boston • San Francisco • Toronto • Sydney • Dubai • Singapore • Hong Kong
Tokyo • Seoul • Taipei • New Delhi • Cape Town • São Paulo • Mexico City • Madrid * Amsterdam • Munich • Paris • Milan

CONTENTS

List of business snapshots.....	15
List of technical notes.....	16
Preface	17
Chapter 1. Introduction.....	23
1.1 Exchange-traded markets.....	24
1.2 Over-the-counter markets	25
1.3 Forward contracts.....	28
1.4 Futures contracts	30
1.5 Options.....	31
1.6 Types of traders.....	33
1.7 Hedgers	34
1.8 Speculators	36
1.9 Arbitrageurs	39
1.10 Dangers.....	39
Summary.....	41
Further reading.....	41
Practice questions.....	42
Chapter 2. Futures markets and central counterparties	46
2.1 Background	46
2.2 Specification of a futures contract.....	48
2.3 Convergence of futures price to spot price	50
2.4 The operation of margin accounts	51
2.5 OTC markets.....	54
2.6 Market quotes	57
2.7 Delivery.....	60
2.8 Types of traders and types of orders.....	61
2.9 Regulation	62
2.10 Accounting and tax	63
2.11 Forward vs. futures contracts.....	64
Summary	65
Further reading.....	66
Practice questions.....	67
Chapters. Hedging strategies using futures	70
3.1 Basic principles	70
3.2 Arguments for and against hedging.....	72
3.3 Basis risk.....	75
3.4 Cross hedging.....	79
3.5 Stock index futures.....	84
3.6 Stack and roll.....	89
Summary.....	90

Further reading.....	92
Practice questions.....	, 93
Appendix: Capital asset pricing model.....	96
Chapter 4. Interest rates.....	98
4.1 Types of rates.....	98
4.2 Reference rates	99
4.3 The risk-free rate	101
4.4 Measuring interest rates	101
4.5 Zero rates.....	104
4.6 Bond pricing.....	105
4.7 Determining zero rates.....	106
4.8 Forward rates	109
4.9 Forward rate agreements.....	110
4.10 Duration.....	112
4.11 Convexity.....	116
4.12 Theories of the term structureof interest rates.....	117
Summary.....	119
Further reading.....	120
Practice questions.....	121
Chapter 5. Determination of forwardand futures prices.....	124
5.1 Investment assets vs. consumption assets.....	124
5.2 Short selling.....	125
5.3 Assumptions and notation.....	126
5.4 Forward price for an investment asset.....	127
5.5 Known income	130
5.6 Known yield.....	132
5.7 Valuing forward contracts	133
5.8 Are forward prices and futures prices equal?	135
5.9 Futures prices of stock indices.....	135
5.10 Forward and futures contracts on currencies	137
5.11 Futures on commodities	141
5.12 The cost of carry.....	143
5.13 Delivery options.....	144
5.14 Futures prices and expected future spot prices.....	144
Summary.....	147
Further reading.....	148
Practice questions.....	149
Chapter 6. Interest rate futures	152
6.1 Day count and quotation conventions	152
6.2 Treasury bond futures.....	155
6.3 Eurodollar and SOFR futures.....	160
6.4 Duration-based hedging strategies using futures.....	165
6.5 Hedging portfolios of assets and liabilities	167
Summary.....	168
Further reading.....	168
Practice questions.....	169
Chapter?. Swaps ..	172
7.1 Mechanics of interest rate swaps.....	172
7.2 Determining risk-free rates.....	175

7.3	Reasons for trading interest rate swaps.....	176
7.4	The organization of trading	178
7.5	The comparative-advantage argument.....	181
7.6	Valuation of interest rate swaps.....	183
7.7	How the value changes through time.....	185
7.8	Fixed-for-fixed currency swaps.....	186
7.9	Valuation of fixed-for-fixed currency swaps.....	190
7.10	Other currency swaps.....	192
7.11	Credit risk.....	193
7.12	Credit default swaps.....	193
7.13	Other types of swaps.....	194
	Summary.....	196
	Further reading.....	196
	Practice questions.....	197
Chapter 8.	Securitization and the financial crisis of 2007-8	201
8.1	Securitization	201
8.2	The U.S. housing market.....	205
8.3	What went wrong?.....	209
8.4	The aftermath	211
	Summary.....	213
	Further reading.....	213
	Practice questions.....	215
Chapter 9.	XVAs.....	216
9.1	CVAandDVA.....	216
9.2	FVAandMVA.....	219
9.3	KVA.....	222
9.4	Calculation issues	223
	Summary.....	224
	Further reading.....	225
	Practice questions.....	, 226
Chapter 10.	Mechanics of options markets.....	227
10.1	Types of options.....	227
10.2	Option positions.....	229
10.3	Underlying assets.....	231
10.4	Specification of stock options	233
10.5	Trading.....	236
10.6	Trading costs	237
10.7	Margin requirements.....	237
10.8	The options clearing corporation.....	239
10.9	Regulation	239
10.10	Taxation	240
10.11	Warrants, employee stock options, and convertibles.....	241
10.12	Over-the-counter options markets.....	242
	Summary.....	243
	Further reading.....	243
	Practice questions.....	244
Chapter 11.	Properties of stock options.....	247
11.1	Factors affecting option prices.....	247
11.2	Assumptions and notation.....	251

11.3	Upper and lower bounds for option prices.....	252
11.4	Put-call parity.....	255
11.5	Calls on a non-dividend-paying stock.....	257
11.6	Puts on a non-dividend-paying stock.....	260
11.7	Effect of dividends.....	262
	Summary.....	263
	Further reading.....	264
	Practice questions.....	265
Chapter 12.	Trading strategies involving options.....	268
12.1	Principal-protected notes	268
12.2	Trading an option and the underlying asset.....	270
12.3	Spreads.....	272
12.4	Combinations.....	280
12.5	Other payoffs.....	283
	Summary.....	284
	Further reading.....	285
	Practice questions.....	285
Chapter 13.	Binomial trees.....	288
13.1	A one-step binomial model and a no-arbitrage argument.....	288
13.2	Risk-neutral valuation.....	292
13.3	Two-step binomial trees.....	294
13.4	A put example.....	297
13.5	American options.....	298
13.6	Deka.....	299
13.7	Matching volatility with u and d	300
13.8	The binomial tree formulas.....	302
13.9	Increasing the number of steps.....	302
13.10	Using D er ivaGem.....	303
13.11	Options on other assets.....	304
	Summary.....	308
	Further reading.....	308
	Practice questions.....	309
	Appendix: Derivation of the Black-Scholes-Merton option-pricing formula from a binomial tree.....	312
Chapter 14.	Wiener processes and Ito's lemma.....	316
14.1	The Markov property.....	316
14.2	Continuous-time stochastic processes	317
14.3	The process for a stock price.....	322
14.4	The parameters.....	325
14.5	Correlated processes.....	326
14.6	Ito's lemma	327
14.7	The lognormal property	328
14.8	Fractional Brownian motion.....	329
	Summary.....	330
	Further reading.....	332
	Practice questions.....	333
	Appendix: A nonrigorous derivation of Ito's lemma.....	336
Chapter 15.	The Black-Scholes-Merton model.....	338
15.1	Lognormal property of stock prices.....	339
15.2	The distribution of the rate of return.....	340

15.3	The expected return	341
15.4	Volatility	342
15.5	The idea underlying the Black-Scholes-Merton differential equation.....	346
15.6	Derivation of the Black-Scholes-Merton differential equation	348
15.7	Risk-neutral valuation.....	351
15.8	Black-Scholes-Merton pricing formulas	352
15.9	Cumulative normal distribution function	355
15.10	Warrants and employee stock options.....	356
15.11	Implied volatilities.....	358
15.12	Dividends.....	360
	Summary.....	363
	Further reading.....	364
	Practice questions.....	365
	Appendix: Proof of the Black-Scholes-Merton formula using risk-neutral valuation.....	369
Chapter 16.	Employee stock options.....	371
16.1	Contractual arrangements.....	371
16.2	Do options align the interests of shareholders and managers?.....	373
16.3	Accounting issues.....	374
16.4	Valuation.....	375
16.5	The backdating scandal	380
	Summary.....	381
	Further reading.....	381
	Practice questions.....	382
Chapter 17.	Options on stock indices and currencies.....	384
17.1	Options on stock indices.....	384
17.2	Currency options	386
17.3	Options on stocks paying known dividend yields.....	389
17.4	Valuation of European stock index options.....	391
17.5	Valuation of European currency options	394
17.6	American options	395
	Summary.....	396
	Further reading.....	397
	Practice questions.....	397
Chapter 18.	Futures options and Black's model.....	401
18.1	Nature of futures options.....	401
18.2	Reasons for the popularity of futures options.....	404
18.3	European spot and futures options.....	404
18.4	Put-call parity.....	405
18.5	Bounds for futures options.....	406
18.6	Drift of a futures price in a risk-neutral world	407
18.7	Black's model for valuing futures options.....	408
18.8	Using Black's model instead of Black-Scholes-Merton	409
18.9	Valuation of futures options using binomial trees.....	410
18.10	American futures options vs. American spot options.....	412
18.11	Futures-style options.....	413
	Summary.....	413
	Further reading.....	414
	Practice questions.....	414

22.3	Model-building approach	521
22.4	The linear model.....	524
22.5	The quadratic model	530
22.6	Monte Carlo simulation.....	533
22.7	Comparison of approaches.....	533
22.8	Back testing.....	534
22.9	Principal components analysis.....	534
	Summary.....	537
	Further reading.....	538
	Practice questions.....	539
Chapter 23.	Estimating volatilities and correlations	542
23.1	Estimating volatility.....	542
23.2	The exponentially weighted moving average model.....	544
23.3	The GARCH(1,1) model	546
23.4	Choosing between the models	547
23.5	Maximum likelihood methods.....	548
23.6	Using GARCH(1,1) to forecast future volatility.....	553
23.7	Correlations.....	556
	Summary.....	558
	Further reading.....	559
	Practice questions.....	559
Chapter 24.	Credit risk.....	562
24.1	Credit ratings.....	562
24.2	Historical default probabilities.....	563
24.3	Recovery rates.....	564
24.4	Estimating default probabilities from bond yield spreads	564
24.5	Comparison of default probability estimates.....	567
24.6	Using equity prices to estimate default probabilities	570
24.7	Credit risk in derivatives transactions	571
24.8	Default correlation	577
24.9	Credit VaR.....	580
	Summary.....	582
	Further reading.....	583
	Practice questions.....	583
Chapter 25.	Credit derivatives.....	587
25.1	Credit default swaps.....	588
25.2	Valuation of credit default swaps	591
25.3	Credit indices	595
25.4	The use of fixed coupons	596
25.5	CDS forwards and options	597
25.6	Basket credit default swaps	597
25.7	Total return swaps	597
25.8	Collateralized debt obligations.....	599
25.9	Role of correlation in a basket CDS and CDO.....	601
25.10	Valuation of a synthetic CDO.....	601
25.11	Alternatives to the standard market model	608
	Summary.....	610
	Further reading.....	610
	Practice questions.....	611

Chapter 26. Exotic options.....	614
26.1 Packages.....	614
26.2 Perpetual American call and put options	615
26.3 Nonstandard American options	616
26.4 Gap options	617
26.5 Forward start options.....	618
26.6 Cliquet options.....	618
26.7 Compound options.....	618
26.8 Chooser options.....	619
26.9 Barrier options.....	620
26.10 Binary options.....	622
26.11 Lookback options	623
26.12 Shout options.....	625
26.13 Asian options	626
26.14 Options to exchange one asset for another.....	627
26.15 Options involving several assets	628
26.16 Volatility and variance swaps.....	629
26.17 Static options replication.....	632
Summary.....	634
Further reading.....	635
Practice questions.....	635
Chapter 27. More on models and numerical procedures.....	640
27.1 Alternatives to Black-Scholes-Merton.....	641
27.2 Stochastic volatility models.....	646
27.3 The IVF model	649
27.4 Convertible bonds.....	650
27.5 Path-dependent derivatives.....	653
27.6 Barrier options.....	656
27.7 Options on two correlated assets.....	658
27.8 Monte Carlo simulation and American options.....	660
Summary.....	665
Further reading.....	666
Practice questions.....	667
Chapter 28. Martingales and measures.....	670
28.1 The market price of risk	671
28.2 Several state variables.....	674
28.3 Martingales.....	675
28.4 Alternative choices for the numeraire.....	676
28.5 Extension to several factors	679
28.6 Black's model revisited	680
28.7 Option to exchange one asset for another.....	681
28.8 Change of numeraire.....	682
Summary.....	684
Further reading.....	685
Practice questions.....	685
Chapter 29. Interest rate derivatives: The standard market models.....	688
29.1 Bond options	688
29.2 Interest rate caps and floors	693
29.3 European swap options.....	699
29.4 Hedging interest rate derivatives.....	703

Summary.....	703
Further reading.....	704
Practice questions.....	704
Chapter 30. Convexity, timing, and quantoadjustments	707
30.1 Convexity adjustments.....	707
30.2 Timing adjustments.....	710
30.3 Quantos.....	711
Summary.....	714
Further reading.....	715
Practice questions.....	715
Appendix: Proof of the convexity adjustment formula	718
Chapter 31. Equilibrium models of the short rate.....	719
31.1 Background	719
31.2 One-factor models.....	721
31.3 Real-world vs. risk-neutral processes.....	726
31.4 Estimating parameters.....	727
31.5 More sophisticated models.....	728
Summary.....	729
Further reading.....	729
Practice questions.....	729
Chapter 32. No-arbitrage models of the short rate	732
32.1 Extensions of equilibrium models.....	732
32.2 Options on bonds.....	736
32.3 Volatility structures.....	737
32.4 Interest rate trees.....	738
32.5 A general tree-building procedure.....	740
32.6 Calibration.....	749
32.7 Hedging using a one-factor model	751
Summary.....	752
Further reading.....	752
Practice questions.....	752
Chapter 33. Modeling forward rates.....	755
33.1 The Heath, Jarrow, and Morton model	755
33.2 The BGM model.....	758
33.3 Agency mortgage-backed securities	768
Summary.....	770
Further reading.....	770
Practice questions.....	771
Chapter 34. Swaps revisited	773
34.1 Variations on the vanilla deal.....	773
34.2 Compounding swaps	775
34.3 Currency and nonstandard swaps	776
34.4 Equity swaps.....	777
34.5 Swaps with embedded options.....	779
34.6 Other swaps.....	781
Summary.....	782
Further reading.....	783
Practice questions.....	783

Chapter 35. Energy and commodity derivatives	785
35.1 Agricultural commodities	785
35.2 Metals.....	786
35.3 Energy products.....	787
35.4 Modeling commodity prices.....	789
35.5 Weather derivatives.....	795
35.6 Insurance derivatives	796
35.7 Pricing weather and insurance derivatives.....	797
35.8 How an energy producer can hedge risks.....	798
Summary.....	799
Further reading.....	799
Practice questions.....	800
Chapter 36. Real options.....	802
36.1 Capital investment appraisal.....	802
36.2 Extension of the risk-neutral valuation framework	803
36.3 Estimating the market price of risk	805
36.4 Application to the valuation of a business	806
36.5 Evaluating options in an investment opportunity.....	806
Summary.....	813
Further reading.....	813
Practice questions.....	814
Chapter 37. Derivatives mishaps and what we can learn from them.....	815
37.1 Lessons for all users of derivatives.....	815
37.2 Lessons for financial institutions.....	819
37.3 Lessons for nonfinancial corporations	824
Summary.....	826
Further reading.....	826
Glossary of terms	827
DerivaGem software.....	851
Exchanges trading futures and options.....	856
Table for $N(x)$ When $x < 0$	857
Author index.....	859
Subject index.....	863