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# Machine Learning in Finance

## From Theory to Practice

4<sup>^</sup> Springer

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## Part I Machine Learning with Cross-Sectional Data

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