

Econometrics

FUMIO HAYASHI

PRINCETON UNIVERSITY PRESS
PRINCETON AND OXFORD

Contents

List of Figures	xvii
Preface ;	xix
1 Finite-Sample Properties of OLS	3
1.1 The Classical Linear Regression Model	3
The Linearity Assumption	4
Matrix Notation	6
The Strict Exogeneity Assumption	7
Implications of Strict Exogeneity	8
Strict Exogeneity in Time-Series Models	9
Other Assumptions of the Model	10
The Classical Regression Model for Random Samples	12
"Fixed" Regressors	13
1.2 The Algebra of Least Squares	15
OLS Minimizes the Sum of Squared Residuals	15
Normal Equations	16
Two Expressions for the OLS Estimator	18
More Concepts and Algebra	18
Influential Analysis (optional)	21
A Note on the Computation of OLS Estimates	23
1.3 Finite-Sample Properties of OLS	27
Finite-Sample Distribution of b	27
Finite-Sample Properties of s^2	30
Estimate of $\text{Var}(b X)$	31
1.4 Hypothesis Testing under Normality	33
Normally Distributed Error Terms	33
Testing Hypotheses about Individual Regression Coefficients	35
Decision Rule for the f -Test	37
Confidence Interval	38

	p-Value	38
	Linear Hypotheses	39
	The F-Test;	40
	A More Convenient Expression for F	42
	f versus F	43
	An Example of a Test Statistic Whose Distribution Depends on X	45
1.5	Relation to Maximum Likelihood	47
	The Maximum Likelihood Principle	47
	Conditional versus Unconditional Likelihood	47
	The Log Likelihood for the Regression Model	48
	ML via Concentrated Likelihood	48
	Cramer-Rao Bound for the Classical Regression Model	49
	The F-Test as a Likelihood Ratio Test	52
	Quasi-Maximum Likelihood	53
1.6	Generalized Least Squares (GLS)	54
	Consequence of Relaxing Assumption 1.4	55
	Efficient Estimation with Known V	55
	A Special Case: Weighted Least Squares (WLS)	58
	Limiting Nature of GLS	58
1.7	Application: Returns to Scale in Electricity Supply	60
	The Electricity Supply Industry	60
	The Data	60
	Why Do We Need Econometrics?	61
	The Cobb-Douglas Technology	62
	How Do We Know Things Are Cobb-Douglas?	63
	Are the OLS Assumptions Satisfied?	64
	Restricted Least Squares	65
	Testing the Homogeneity of the Cost Function	65
	Detour: A Cautionary Note on R^2	67
	Testing Constant Returns to Scale	67
	Importance of Plotting Residuals	68
	Subsequent Developments	68
	Problem Set	71
	Answers to Selected Questions	84
2	Large-Sample Theory	88
2.1	Review of Limit Theorems for Sequences of Random Variables	88
	Various Modes of Convergence	89
	Three Useful Results	92

Viewing Estimators as Sequences of Random Variables	94
Laws of Large Numbers and Central Limit Theorems	95
2.2 Fundamental Concepts in Time-Series Analysis	97
Need for Ergodic Stationarity	97
Various Classes of Stochastic Processes	98
Different Formulation of Lack of Serial Dependence	106
The CLT for Ergodic Stationary Martingale Differences Sequences	106
2.3 Large-Sample Distribution of the OLS Estimator	109
The Model	109
Asymptotic Distribution of the OLS Estimator	113
s^2 Is Consistent .	115
2.4 Hypothesis Testing	117
Testing Linear Hypotheses	117
• The Test Is Consistent-	119
Asymptotic Power	120
Testing Nonlinear Hypotheses	121
2.5 Estimating $E(e x, X;)$ Consistently	123
Using Residuals for the Errors	123
Data Matrix Representation of S	125
Finite-Sample Considerations	125
2.6 Implications of Conditional Homoskedasticity	126
Conditional versus Unconditional Homoskedasticity	126
Reduction to Finite-Sample Formulas	127
Large-Sample Distribution of t and F Statistics	128
Variations of Asymptotic Tests under Conditional Homoskedasticity	129
2.7 Testing Conditional Homoskedasticity	131
2.8 Estimation with Parameterized Conditional Heteroskedasticity (optional)	133
The Functional Form	133
WLS with Known a	134
Regression of e^2 on z , Provides a Consistent Estimate of a	135
WLS with Estimated a	136
OLS versus WLS	137
2.9 Least Squares Projection	137
Optimally Predicting the Value of the Dependent Variable	138
Best. Linear Predictor	139
OLS Consistently Estimates the Projection Coefficients	. 140

2.10	Testing for Serial Correlation	141
	Box-Pierce and Ljung-Box	142
	Sample Autocorrelations Calculated from Residuals	144
	Testing with Predetermined, but Not Strictly Exogenous, Regressors	146
	An Auxiliary Regression-Based Test	147
2.11	Application: Rational Expectations Econometrics	150
	The Efficient Market Hypotheses	150
	Testable Implications	152
	Testing for Serial Correlation	153
	Is the Nominal Interest Rate the Optimal Predictor?	156
	R , Is Not Strictly Exogenous	158
	Subsequent Developments	159
2.12	Time Regressions	160
	The Asymptotic Distribution of the OLS Estimator	161
	Hypothesis Testing for Time Regressions	163
	Appendix 2.A: Asymptotics with Fixed Regressors	164
	Appendix 2.B: Proof of Proposition 2.10	165
	Problem Set	168
	Answers to Selected Questions	183
3	Single-Equation $GMV1$	186
3.1	Endogeneity Bias: Working's Example	187
	A Simultaneous Equations Model of Market Equilibrium	187
	Endogeneity Bias	188
	Observable Supply Shifters	189
3.2	More Examples	193
	A Simple Macroeconometric Model	193
	Errors-in-Variables	194
	Production Function	196
3.3	The General Formulation	198
	Regressors and Instruments	198
	Identification	200
	Order Condition for Identification	202
	The Assumption for Asymptotic Normality	202
3.4	Generalized Method of Moments Defined	204
	Method of Moments	205
	Generalized Method of Moments	206
	Sampling Error	207

3.5	Large-Sample Properties of GMM	208
	Asymptotic Distribution of the GMM Estimator,	209
	Estimation of Error Variance	210
	Hypothesis Testing	211
	Estimation of S	212
	Efficient GMM Estimator	212
	Asymptotic Power	214
	Small-Sample Properties	215
3.6	Testing Overidentifying Restrictions	217
	Testing Subsets of Orthogonality Conditions	218
3.7	Hypothesis Testing by the Likelihood-Ratio Principle	222
	The LR Statistic ¹ for the Regression Model	223
	Variable Addition Test (optional)	224
3.8	Implications of Conditional Homoskedasticity	225
	Efficient GMM Becomes 2SLS	226
	J Becomes Sargan's Statistic	227
	Small-Sample Properties of 2SLS	229
	Alternative Derivations of 2SLS	229
	When Regressors Are Predetermined	231
	Testing a Subset of Orthogonality Conditions	232
	Testing Conditional Homoskedasticity	234
	Testing for Serial Correlation	234
3.9	Application: Returns from Schooling	236
	The NLS-Y Data	236
	The Semi-Log Wage Equation	237
	Omitted Variable Bias	238
	IQ as the Measure of Ability	239
	Errors-in-Variables	239
	2SLS to Correct for the Bias	242
	Subsequent Developments	243
	Problem Set	244
	Answers to Selected Questions	254
4	Multiple-Equation GMM	258
4.1	The Multiple-Equation Model	259
	Linearity	259
	Stationarity and Ergodicity	260
	Orthogonality Conditions	261
	Identification	262

The Assumption for Asymptotic Normality	264
Connection to the "Complete" System of Simultaneous Equations	265
4.2 Multiple-Equation GMM Defined	265
4.3 Large-Sample Theory	268
4.4 Single-Equation versus Multiple-Equation Estimation	271
When Are They "Equivalent"?	272
Joint Estimation Can Be Hazardous	273
4.5 Special Cases of Multiple-Equation GMM: FIVE, 3SLS, and SUR	274
Conditional Homoskedasticity	274
Full-Information Instrumental Variables Efficient (FIVE)	275
Three-Stage Least Squares (3SLS)	276
Seemingly Unrelated Regressions (SUR)	279
SUR versus OLS	281
4.6 Common Coefficients	286
The Model with Common Coefficients	286
The GMM Estimator	287
Imposing Conditional Homoskedasticity	288
Pooled OLS	290
Beautifying the Formulas	292
The Restriction That Isn't	293
4.7 Application: Interrelated Factor Demands	296
The Translog Cost Function	296
Factor Shares	297
Substitution Elasticities	298
Properties of Cost Functions	299
Stochastic Specifications	300
The Nature of Restrictions	301
Multivariate Regression Subject to Cross-Equation Restrictions	302
Which Equation to Delete?	304
Results	305
Problem Set	308
Answers to Selected Questions	320
Panel Data	323
5.1 The Error-Components Model	324
Error Components	324
Group Means	327
A Reparameterization	327
5.2 The Fixed-Effects Estimator	330

The Formula	-	330
Large-Sample Properties		331
Digression: When \neq , Is Spherical		333
Random Effects versus Fixed Effects		334
Relaxing Conditional Homoskedasticity		335
5.3 Unbalanced Panels (optional)		337
"Zeroing Out" Missing Observations		338
-Zeroing Out versus Compression		339
No Selectivity Bias		340
5.4 Application: International Differences in Growth Rates		342
Derivation of the Estimation Equation		342
Appending the Error Term		343
Treatment of a ,		344
Consistent Estimation of Speed of Convergence		345
Appendix 5.A: Distribution of Hausman Statistic		346
Problem Set		349
Answers to Selected Questions		363
6 Serial Correlation		365
6.1 Modeling Serial Correlation: Linear Processes		365
MA(∞)		366
MA(∞) as a Mean Square Limit		366
Filters		369
Inverting Lag Polynomials		372
6.2 ARMA Processes		375
AR(1) and Its MA(∞) Representation		376
Autocovariances of AR(1)		378
AR(p) and Its MA(∞) Representation		378
ARMA(p, q)		380
ARMA(p, q) with Common Roots		382
Invertibility		383
Autocovariance-Generating Function and the Spectrum		383
6.3 Vector Processes		387
6.4 Estimating Autoregressions		392
Estimation of AR(1)		392
Estimation of AR(p)		393
Choice of Lag Length		394
Estimation of VARs		397
Estimation of ARMA(p, q)		398

6.5	Asymptotics for Sample Means of Serially Correlated Processes	400
	LLN for Covariance-Stationary Processes	401
	Two Central Limit Theorems	402
	Multivariate Extension	404
6.6	Incorporating Serial Correlation in GMM	406
	The Model and Asymptotic Results	406
	Estimating S When Autocovariances Vanish after Finite Lags	407
	Using Kernels to Estimate S	408
	VARHAC	410
6.7	Estimation under Conditional Homoskedasticity (Optional)	413
	Kernel-Based Estimation of S under Conditional Homoskedasticity	413
	Data Matrix Representation of Estimated Long-Run Variance	414
	Relation to GLS	415
6.8	Application: Forward Exchange Rates as Optimal Predictors	418
	The Market Efficiency Hypothesis	419
	Testing Whether the Unconditional Mean Is Zero	420
	Regression Tests	423
	Problem Set	428
	Answers to Selected Questions	441
7	Extremum Estimators	445
7.1	Extremum Estimators	446
	"Measurability" of θ	446
	Two Classes of Extremum Estimators	447
	Maximum Likelihood (ML)	448
	Conditional Maximum Likelihood	450
	Invariance of ML	452
	Nonlinear Least Squares (NLS)	453
	Linear and Nonlinear GMM	454
7.2	Consistency	456
	Two Consistency Theorems for Extremum Estimators	456
	Consistency of M-Estimators	458
	Concavity after Reparameterization	461
	Identification in NLS and ML	462
	Consistency of GMM	467
7.3	Asymptotic Normality	469
	Asymptotic Normality of M-Estimators	470
	Consistent Asymptotic Variance Estimation	473
	Asymptotic Normality of Conditional ML	474

Two Examples	476
Asymptotic Normality of GMM	478
GMM versus ML	481
Expressing the Sampling Error in a Common Format	483
7.4 Hypothesis Testing	487
The Null Hypothesis	487
The Working Assumptions	489
The Wald Statistic	489
The Lagrange Multiplier (LM) Statistic	491
The Likelihood Ratio (LR) Statistic	493
Summary of the Trinity	494
• 7.5 Numerical Optimization	497
Newton-Raphson	497
Gauss-Newton	498
Writing Newton-Raphson and Gauss-Newton in a Common Format	498
Equations Nonlinear in Parameters Only	499
Problem Set	501
Answers to Selected Questions	505
8 Examples of Maximum Likelihood	507
8.1 Qualitative Response (QR) Models	507
Score and Hessian for Observation t	508
Consistency	509
Asymptotic Normality	510
8.2 Truncated Regression Models	511
The Model	511
Truncated Distributions	512
The Likelihood Function	513
Reparameterizing the Likelihood Function	514
Verifying Consistency and Asymptotic Normality	515
Recovering Original Parameters	517
8.3 Censored Regression (Tobit) Models	518
Tobit Likelihood Function	518
Reparameterization	519
8.4 Multivariate Regressions	521
The Multivariate Regression Model Restated	522
The Likelihood Function	523
Maximizing the Likelihood Function	524

	Consistency and Asymptotic-Normality	525
8.5	FIML	526
	The Multiple-Equation Model with Common Instruments Restated	526
	The Complete System of Simultaneous Equations	529
	Relationship between (T_0, B_0) and δ_0	530
	The FIML Likelihood Function	531
	The FIML Concentrated Likelihood Function	532
	Testing Overidentifying Restrictions	533
	Properties of the FIML Estimator	533
	ML Estimation of the SUR Model	535
8.6	LIML	538
	LIML Defined	538
	Computation of LIML	540
	LIML versus 2SLS	542
8.7	Serially Correlated Observations	543
	Two Questions	543
	Unconditional ML for Dependent Observations	545
	ML Estimation of AR(1) Processes	546
	Conditional ML Estimation of AR(1) Processes	547
	Conditional ML Estimation of $AR(p)$ and $VAR(p)$ Processes	549
	Problem Set	551
9	Unit-Root Econometrics	557
9.1	Modeling Trends	557
	Integrated Processes	558
	Why Is It Important to Know if the Process Is $I(1)$?	560
	Which Should Be Taken as the Null, $I(0)$ or $I(1)$?	562
	Other Approaches to Modeling Trends	563
9.2	Tools for Unit-Root Econometrics	563
	Linear $I(0)$ Processes	563
	Approximating $I(1)$ by a Random Walk	564
	Relation to ARMA Models	566
	The Wiener Process	567
	A Useful Lemma	570
9.3	Dickey-Fuller Tests	573
	The AR(1) Model	573
	Deriving the Limiting Distribution under the $I(1)$ Null	574
	Incorporating the Intercept	577
	Incorporating Time Trend	581

9.4	Augmented Dickey-Fuller Tests	585
	The Augmented Autoregression	585
	Limiting Distribution of the OLS Estimator	586
	Deriving Test Statistics	590
	Testing Hypotheses about f	591
	What to Do When p Is Unknown?	592
	A Suggestion for the Choice of $p_{max}(T)$	594
	Including the Intercept in the Regression	595
	Incorporating Time Trend	597
	Summary of the DF and ADF Tests and Other Unit-Root Tests	599
9.5	Which Unif-Root Test to Use?	601
	Local-to-Unity Asymptotics	602
	Small-Sample Properties	602
9.6	Application: Purchasing Power Parity	603
	The Embarrassing Resiliency of the Random Walk Model?	604
	Problem Set	605
	Answers to Selected Questions	619
10	Cointegration	623
10.1	Cointegrated Systems	624
	Linear Vector $I(0)$ and $I(1)$ Processes	624
	The Beveridge-Nelson Decomposition	627
	Cointegration Defined	629
10.2	Alternative Representations of Cointegrated Systems	633
	Phillips's Triangular Representation	633
	VAR and Cointegration	636
	The Vector Error-Correction Model (VECM)	638
	Johansen's ML Procedure	640
10.3	Testing the Null of No Cointegration	643
	Spurious Regressions	643
	The Residual-Based Test for Cointegration	644
	Testing the Null of Cointegration	649
10.4	Inference on Cointegrating Vectors	650
	The SOLS Estimator	650
	The Bivariate Example	652
	Continuing with the Bivariate Example	653
	Allowing for Serial Correlation	654
	General Case	657
	Other Estimators and Finite-Sample Properties	658

10.5 Application: The Demand for Money in the United States	659
The Data	660
$(m - p, y, R)$ as a Cointegrated System	660
DOLS	662
Unstable Money Demand?	663
Problem Set	665
Appendix A: Partitioned Matrices and Kronecker Products	670
Addition and Multiplication of Partitioned Matrices	671
Inverting Partitioned Matrices	672
Index	675