THIRD EDITION

# Jum C. Nunnally

Late Professor of Psychology Vanderbilt University

## Ira H. Bernstein

Professor of Psychology The University of Texas at Arlington

McGRAW-HILL, INC.

New York St. Louis San Francisco Auckland Bogota Caracas Lisbon London Madrid Mexico City Milan Montreal New Delhi San Juan Singapore Sydney Tokyo Toronto

# CONTENTS

PREFACE

## PART 1 INTRODUCTION

1	Introduction	3
	CHAPTER OVERVIEW	3
	MEASUREMENT IN SCIENCE	3
	What Is "Meaningful" and "Useful"?	
	ADVANTAGES OF STANDARDIZED MEASURES	6
	Objectivity / Quantification / Communication / Economy / Scientific	
	Generalization	
	MEASUREMENT AND MATHEMATICS	8
	Measurement and Statistics MEASUREMENT SCALES	11
	Nominal scales / Ordinal scales / Interval scales / Ratio Scales /	11
	Other Scales / Invariance	
	DECISIONS ABOUT MEASUREMENT SCALES	20
	Ostensive Characteristics / Consequences of Assumptions / Convention	-
1	/Classification as Measurement	
	RECENT TRENDS IN MEASUREMENT	27
	The Impact of Computers / Closed versus Open-Form Solutions /	
	Computer Simulation	
	SUMMARY	29
	SUGGESTED ADDITIONAL READINGS	30
		21
PARI 2	STATISTICAL FOUNDATIONS	31

 2
 Traditional Approaches To Scaling
 33

 CHAPTER OVERVIEW
 33

ix

1

	DATA MATRICES	34
	More Complex Organizations / "Holes" in the Matrix (Missing Data) EVALUATION OF MODELS	36
	Scaling Stimuli versus Scaling People	30
	A BRIEF INTRODUCTION TO PSYCHOPHYSICS	39
	Psychophysical Methods / Absolute Thresholds / Simulating a	00
	Threshold / Difference Thresholds / The Weber Fraction, Fechner's	
	Law, and Psychophysical Scaling / Direct Psychophysics and the	
	Plateau-Stevens Tradition / The Fullerton-Cattell Law / Signal	
	Detection Theory and Modern Psychophysics	
	TYPES OF STIMULI AND RESPONSES	50
	Judgments versus Sentiments / Absolute versus Comparative	
	Responses / Preferences versus Similarity Responses / Specified	
	versus Unspecified Attributes	
	METHODS FOR CONVERTING RESPONSES TO STIMULUS SCALES	53
	Ordinal Methods / Interval Methods / Ratio Methods	
	MODELS FOR SCALING STIMULI	55
	Direct (Subjective Estimate) Models / Indirect (Discriminant) Models /	
	Simulating Thurstone Scaling / A Comparison of the Two	
	Simulations / The Logistic Distribution and Luce's Choice Theory /	
	Averages as Scale Values / Checks and Balances / Multi-item	
	Measures / Item Trace Lines (Item Characteristics Curves) /	
	Difficulty and Discrimination DETERMINISTIC MODELS FOR SCALING PEOPLE	71
		71
	The Guttman Scale / Evaluation of the Guttman Scale PROBABILISTIC MODELS FOR SCALING PEOPLE	75
	Nonmonotone Models / Monotone Models with Specified Distribution	75
	Forms / Monotone Models with Unspecified Distribution Forms	
	SUMMARY	81
	SUGGESTED ADDITIONAL READINGS	82
3	Validity	83
	CHAPTER OVERVIEW	83
	GENERAL CONSIDERATIONS	84
	CONSTRUCT VALIDITY	84
	Domain of Observables / Relations among Observables / Relations	
	among Constructs / Campbell and Fiske's Contribution to Construct	
	Validation	
	PREDICTIVE VALIDITY	94
	The Temporal Relation between Predictor and Criterion / The Criterion Problem / Other Problems in Prediction / The "Composite Criterion" / Validity Coefficients / Validity Generalization / Meta-analysis	
	CONTENT VALIDITY	101
	EXPLICATION OF CONSTRUCTS	104

	Changing Substantive Theories versus Changing Measurement	
	Theories / A Commonsense Point of View	400
	OTHER ISSUES CONCERNING VALIDITY Relations among the Three Types of Validity / Other Names / The	108
	Place of Factor Analysis SUMMARY	112
	SUGGESTED ADDITIONAL READINGS	113
4	Elements of Statistical Description and Estimation	114
	CHAPTER OVERVIEW	114
	CONTINUOUS VERSUS DISCRETE (CATEGORICAL) VARIABLES VARIANCE	115 116
	Transformations of Distributions	
	CORRELATION AND COVARIANCE AS CONCEPTS	120
	THE PEARSON PRODUCT-MOMENT CORRELATION	120
	The Meaning of Pearson Product-Moment Correlation / Computer Applications / Covariance / Other Measures of Linear Relation / Three Special Cases	
	ESTIMATES OF r	125
	Biserial $r(r_{Us})$ / Tetrachoric Correlation ( $r_{iet}$ ) and Related Estimates	
	PEARSON r VERSUS ESTIMATES OF PEARSON r Some Related Issues in Categorization	127
	ASSUMPTIONS UNDERLYING r	129
	FACTORS INFLUENCING r	130
	Restriction of Range / Distribution Form	
	A UNIVERSAL MEASURE OF RELATIONSHIP	135
	PREDICTION, REGRESSION, AND STRUCTURAL EQUATIONS	139
	Regression / Regression Based upon Raw Scores / The Standard Error of Estimate / Partitioning of Variance / Structural Equations	
	STATISTICAL ESTIMATION AND STATISTICAL DECISION THEORY Generalized Least-Squares Estimation / Maximum Likelihood	147
	Estimation / Maximum Likelihood and the Testing of Hierarchical	
	Models / Bayesian Estimation / The Method of Moments /.Equal Weighting (The "It Don't Make No Nevermind Principle") / Properties of Estimators	
	SUMMARY	156
	SUMINIAR T SUGGESTED ADDITIONAL READINGS	158
5		
	Multiple Regression	159
	CHAPTER OVERVIEW	159
	VARIANCES OF LINEAR COMBINATIONS	160
	Variance of a Weighted Sum / Variance of a Sum of Standard Scores /	

Variance of Sums of Dichotomous Distributions / Numerical	
Examples	
CHARACTERISTICS OF SCORE DISTRIBUTIONS	166
Variances / Distribution Shape	
COVARIANCE OF LINEAR COMBINATIONS	171
Correlation of Linear Combinations / Numerical Example	
PARTIAL CORRELATION	175
An Example of Partialling / Higher-Order Partialling / Another Form of	
Partialling	
MULTIPLE CORRELATION AND MULTIPLE REGRESSION	182
The Two-Predictor Case / Numerical Example / The General Case /	
Testing the Significance of ft and Increments in $R$ I Determinants of	
R I Categorical Predictors / Multicollinearity / Predictor Importance	
SELECTION AND ALTERNATIVE WEIGHTINGS OF PREDICTORS	193
Stepwise Inclusion of Predictors / AN Possible Subsets Approaches /	
Hierarchical Inclusion of Variables / Combining Strategies /	
Moderated Multiple Regression / Variable Weighting	
RELATED TOPICS	203
The Analysis of Covariance / Nonlinear Relations / Residual	
Analysis / Canonical Analysis	
SUMMARY	205
SUGGESTED ADDITIONAL READINGS	208
	200

### PART 3 CONSTRUCTION OF MULTI-ITEM MEASURES 209

6	The Theory of Measurement Error	21 <b>1</b>
	CHAPTER OVERVIEW	211
	THE CONCEPT OF MEASUREMENT ERROR	212
	ONE FORM OF CLASSICAL TEST THEORY	215
	THE DOMAIN-SAMPLING MODEL	216
1	Multi-item Measures / Estimates of Reliability / The Importance of the	
	Reliability Coefficient	
	THE MODEL OF PARALLEL TESTS	223
	PERSPECTIVES ON THE TWO MODELS	226
	Factorial Composition	
	PRECISION OF RELIABILITY ESTIMATES	228
	Variances of Items	
	FURTHER DEDUCTIONS FROM THE DOMAIN-SAMPLING MODEL	230
	Test Length / The Reliability of an Item Sample and Coefficient Alpha /	
	Numerical Example / Variance of True and Error Scores /	
	Estimation of True Scores / The Standard Error of Measurement /	
	Attenuation	

	ALTERNATIVE MODELS Factorial Domain-Sampling Model / The Binomial Model	241
	RELIABILITY AS STABILITY OVER TIME	243
	Difference Scores / One Other Consideration SUMMARY	246
	SUGGESTED ADDITIONAL READINGS	247
7	The Assessment of Reliability	248
	CHAPTER OVERVIEW	248
	SOURCES OF ERROR Variation within a Test / Variation between Tests	249
	ESTIMATION OF RELIABILITY	251
	Internal Consistency / Alternative Forms / Other Estimates of Reliability / Long-Range Stability	201
	USES OF THE RELIABILITY COEFFICIENT	256
	Corrections for Attenuation / Confidence Intervals / Effect of	
	Dispersion on Reliability	000
	MAKING MEASURES RELIABLE Test Length / Standards of Reliability / Limitations on the Reliability Coefficient's Utility	262
	RELIABILITY OF LINEAR COMBINATIONS	266
	Negative Elements / Weighted Sums / Principles Concerning the Reliability of Linear Combinations	
	AN ANALYSIS OF VARIANCE APPROACH TO RELIABILITY	274
	Some Basic Concepts / Application to the Study of Reliability GENERALIZABILITY THEORY	279
	Basic Concepts / Generalizability Studies and Decision Studies / A Single Facet Design / Theoretical Considerations / Applying the Results of a Single Facet <i>G</i> Study to a <i>D</i> Study / A Fixed-Facet Design / Higher-Order Designs	
	SUMMARY	290
	SUGGESTED ADDITIONAL READINGS	292
8	Construction of Conventional Tests	293
	CHAPTER OVERVIEW	293
	CONSTRUCTION OF TESTS DESIGNED FOR CONTENT VALIDATION The Domain of Content and Test Plan / Test Items / Test Length / Sample of Subjects / Item Analysis / Item Selection / Norms / The Role of External Correlates	295
	CONSTRUCTION OF TESTS DESIGNED FOR CONSTRUCT VALIDATION The Hypothesis and Domain of Content / Content Homogeneity / Methodological Heterogeneity / Relations among Measures and Constructs / The Role of Factor Analysis / Item Analysis and Selection / The Inadequacy of Rational Approaches to Test	310

	Construction / The Inadequacy of Empirical (Criterion-Oriented) Approaches to Test Construction / Norms / Applying the Measure / Some Examples of Constructs in the Abilities Area CONSTRUCTION OF TESTS DESIGNED FOR PREDICTIVE	
	VALIDATION	324
	Item Analysis, Item Selection, and Norms PROBLEMS UNIQUE TO CERTAIN TESTING SITUATIONS	326
	Reversing the Direction of Keying / Unipolar versus Bipolar Attributes / Discrimination at a Point / Equidiscriminating Tests / Weighting of Items / Taking Advantage of Chance	
	SUMMARY •	334
	SUGGESTED ADDITIONAL READINGS	337
9	Special Problems in Classical Test Theory	338
	CHAPTER OVERVIEW	338
	GUESSING	340
	The Blind Guessing Model and Abbott's Formula / Effectsof Guessing on Test Parameters / The Accuracy of the Correction for Blind Guessing/Sophisticated Guessing Models / Practical Considerations / Using the Model to Estimate Test Parameters / Multiple-Choice versus Short Answer Tests	
	SPEED TESTS	348
	The Internal Structure of Speed Tests / The Item Pool / Measurement of Reliability / Factor Composition / Variables Relating to Speed / Statistical Effects of Time Limits / One-Trial Measures of the Effects of Time Limits / Correction for Guessing in Speed Tests / Timed- Power Tests / Speed-Difficulty Tests / Factors Measured by Speed and Power Tests / Implications ADVERSE IMPACT, IMPROPER DISCRIMINATION, TEST	
	BIAS, AND DISPARITY	357
	Definitions of Bias / Disparity and Bias / Test Bias, Regression, and the Clean/ Rule / Applying Linear Regression to Salary Disputes / Reverse Regression / Residual Analysis / Simpson's Paradox Revisited / Bias in Content-Validated Measures / Barriers and Cutoffs / Selection Fairness and Quotas / Pooled versus Separate Group Norms	
	HALO EFFECTS	373
	Traditional Measures of Halo / Recent Developments in the Study of Halo	
	RESPONSE BIASES AND RESPONSE STYLES Sources of Bias / Changes in Test Scores as Personality Changes / Carelessness and Confusion / The Role of Social Desirability / Other Proposed Stylistic Variables	376

	MULTISCALE TESTS	386
	Item Overlap	
	SUMMARY SUGGESTED ADDITIONAL READINGS	388 392
10	Recent Developments in Test Theory	393
	CHAPTER OVERVIEW ITEM RESPONSE THEORY Conditional Independence / One-Parameter Models / Two-Parameter Models / Three-Parameter Models / Item and Test Information / The Bock Nominal Model / The Samejima Model for Graded (Ordinal) Responses / A Nonparametric Approach / Other IRT Models / Applications to Nonstandard Testing Conditions / Scoring Algorithms	393 394
	DIFFERENTIAL ITEM FUNCTIONING (ITEM BIAS) A Substantive Example / A Simulated Example / Differential Alternative Functioning / IRT Approaches to Assessing DIF / Alternative IRT Approaches / Classical Approaches to Assessing DIF / Content Bias TAILORED TESTS AND COMPUTERIZED ADAPTIVE	416
	TESTING Tailored Testing and Psychophysical Thresholds / Applying the Staircase Principle to Psychometrics / Flexilevel Tests / More Complex Forms of Tailored Testing / Perspectives on Tailored Tests	428
	COMMENTARY ON IRT	433
	ACHIEVEMENT TESTS FOR MASTERY LEARNING Nature of Mastery Learning / Test Construction / Definition of "Mastery" / Practical Problems	435
	SUMMARY	438
	SUGGESTED ADDITIONAL READINGS	442
PART 4	FACTOR ANALYSIS	445
11	Factor Analysis I: The General Model and Variance	
	Condensation	447
	CHAPTER OVERVIEW	447
	USES OF FACTOR ANALYSIS Factors as Groupings of Variables / Exploratory and Confirmatory Analysis / Factor Analysis and Scientific Generalization / Variable and Subject Selection	449

#### BASIC CONCEPTS

The General Factor Model / The Unit of Measurement / Estimating

454

	Correlations / Structure Elements / Successive versus	
	Simultaneous Factoring / Geometric and Algebraic Interpretations /	
	Components of Variance / Types of Factors	400
	CONDENSING VARIANCE IN EXPLORATORY FACTOR ANALYSIS	468
	The Role of the Correlation Matrix / Properties of a Factor Solution	
	CENTROID CONDENSATION	472
	PRINCIPAL COMPONENT AND PRINCIPAL AXIS	
	CONDENSATION	473
	Principal Components / Mathematical Properties of Principal	
	Components / Principal Axis Solutions	
	MAXIMUM LIKELIHOOD AND RELATED FORMS OF CONDENSATION	477
	"• Usefulness of ML Factoring / Variants on ML Factoring	
	OTHER METHODS OF CONDENSATION	481
	DETERMINING THE NUMBER OF FACTORS	482
	Consequences of Choosing a Given Number of Factors	
	CAUSAL INDICATORS	484
	SUMMARY	485
	SUGGESTED ADDITIONAL READINGS	490
12	Exploratory Factor Analysis II: Rotation and Other Topics	491
	CHAPTER OVERVIEW	491
	FACTOR ROTATION	493
	Geometric Analogy / Visual Rotation / Further Mathematics of Rotation	
	/ Oblique Rotations / Simple and "Simpler" Structures / Reference	
	Vectors	
	ANALYTIC ROTATIONS	505
	Quartimax /Varimax / Promax	
	ESTIMATION OF FACTOR SCORES	507
	Practical Considerations in Obtaining Factor Scores	001
	RELATIONS AMONG THE VARIOUS MATRICES	512
	THE COMMON FACTOR MODEL	514
	The Problem of Communality Estimation / Factorable Matrices / Matrix	014
	Rank / Unities as Communality Estimates / Communalities Derived	
,	from Hypotheses / Statistical Criteria of Rank / Iteration / Squared	
	Multiple Correlations / Reliability Coefficients / Direct Estimation /	
	Some Major Differences between Component and Common Factor	
	Solutions / Some Conceptual Problems with the Common Factor	
	Model / Effects of Number of Variables and Average Correlation upon	
	the Factor Structure FACTOR ANALYTIC DESIGNS	500
		526
	Alternative Designs / Three-Mode Factor Analysis	504
	AD-LIB FACTORING	531
	HIGHER-ORDER FACTORS	532
	HOW TO FOOL YOURSELF WITH FACTOR ANALYSIS	533

	AN OUTLOOK ON EXPLORATORY FACTOR ANALYSIS	535
	Practical Considerations in Factor Analysis	
	SUMMARY	536
	SUGGESTED ADDITIONAL READINGS	541
13	Confirmatory Factor Analysis	542
	CHAPTER OVERVIEW	542
	SPEARMAN'S GENERAL FACTOR SOLUTION	544
	COMPARING FACTORS IN DIFFERENT ANALYSES	548
	Classical Approaches to Testing Factor Invariance / Some Practical	
	Aspects of Comparing Factor Structures / Comparing Overall	
	Solutions / ACS Approaches	
	TESTING WEAK THEORIES (THEORIES CONCERNING GROUPINGS OF	
	VARIABLES)	554
	Multiple Group Confirmatory Analysis / Procrustes	
	Confirmatory Analysis/ACS Confirmatory Analysis / A Comparison	
	of the Three Approaches FACTORING CATEGORICAL VARIABLES (ITEM LEVEL FACTORING)	570
	ACS and Related Approaches with PM Measures / Multiscale Analyses	570
	TESTING STRONG THEORIES	576
	Introduction to the Full ACS Model / ACS Notation / Assumptions /	
	Properties of Path Coefficients / A Note on Inferring Causality from	
	Correlational Data / Model Specification in ACS / Recursive and	
	Nonrecursive Models / Cross-lagged Correlation / Applying ACS /	
	Reapplying ACS / Classical Approaches to Strong Theories	
	SUMMARY	590
	SUGGESTED ADDITIONAL READINGS	594
PART 5	ADDITIONAL STATISTICAL MODELS, CONCEPTS, AND ISSUES	595
1,4	Profile Analysis, Discriminant Analysis, and Multidimensional Scaling	597
	CHAPTER OVERVIEW	597
	CLUSTER ANALYSIS	598
	PROBLEMS IN PROFILE ANALYSIS	599
	Characteristics of Score Profiles	
	CLUSTERING OF PROFILES	601
	Measures of Profile Similarity / Distance Measure / Hierarchical and	
	Overlapping Clustering	004
	RAW-SCORE FACTOR ANALYSIS	604
	An Example of Raw-Score Factor Analysis / How Raw-Score Factor Analysis Works / Transformations of Variables / Transformations of	
	Profiles	
	Tonics	

	DISCRIMINANT ANALYSIS	610
	Geometric Interpretation of Discriminant Analysis / Linear Discriminant Function / Multiple Linear Discriminant Functions / Placement / Evaluation of Discriminant Analysis	
	PATTERN ANALYSIS	620
	Discovering Latent Groups / Discriminating among Existing Groups / Evaluation of Pattern Analysis	
	MULTIDIMENSIONAL SCALING .	621
	<ul> <li>Spatial Conceptions of MDS / An Overview of Alternative Approaches to MDS / Psychophysical Methods Based upon Similarity / Psychophysical Methods Based upon Attribute Ratings / Indirect Methods / Vector-Space Ratio Methods / Euclidian Distance Ratio Methods / Interval Methods / Ordinal Methods and ALSCAL / Some Empirical Properties of Alternative MDS Solutions / MDS of Correlation Matrices / Scaling of Individual Differences / An Example of the Use of MDS / Some Concluding Comments</li> </ul>	
	DOMINANCE (PREFERENCE) SCALING	645
	The Unfolding Concept / Multidimensional Unfolding and ALSCAL	
	SUMMARY	648
	SUGGESTED ADDITIONAL READINGS	651
15	The Analysis of Categorical Data, Binary Classification, and Alternatives to	
	Geometric Representations	652
	CHAPTER OVERVIEW "	652
	CATEGORICAL MODELING	654
	<ul> <li>Two-way Independence / Association (Nonindependence) / Alternative Models for the 2 x 2 Case / Measures of Association in the 2 x 2 Design / More about C3<sup>2</sup> / The Generalized Logit Variant / Structural and Random Zeros / Multiple Levels on a Variable / Higher-Order Designs / Predictor-Criterion Models / Multiple Response Categories in Predictor-Criterion Models/Some Important Assumptions / Log-linear Modeling and Item Response Theory / More-Specific Categorical Models / Logistic Regression / Comparing Groups with Logistic Regression / An Illustrative Problem / A Note on Residuals / Predicting Categorical Criteria</li> </ul>	
	BINARY CLASSIFICATION	680
	Classical Signal Detection / Categorical Modeling Approaches to the Equal Variance Gaussian Model / General Recognition Theory / Application to Condensation Tasks / MDS, Dissimilarity Judgments, and General Recognition Theory / Implications for Measurement	
	NONGEOMETRIC AND NON-EUCLIDIAN MODELS	696
	Nearest Neighbors / Tree Representations / Network and Graph Theoretic Approaches / Conclusions	

SUMMARY	704
SUGGESTED ADDITIONAL READINGS	708
REFERENCES	709
INDEXES	
Name Index	735
Subject Index	741