

CHAPMAN & HALL/CRC FINANCIAL MATHEMATICS SERIES

Robust Libor Modelling and Pricing of Derivative Products

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