

Antoon Pelsser

# Efficient Methods for Valuing Interest Rate Derivatives

UNIVERSITÄT  
LIECHTENSTEIN

Bibliothek



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