

Credit Derivatives Pricing Models

Models, Pricing and Implementation

Philipp J. Schonbucher



Contents

Preface	xi
Acknowledgements	xv
Abbreviations	xvii
Notation	xix
1 Introduction	1
1.1 The world of credit risk	1
1.2 The components of credit risk	2
1.3 Market structure	4
2 Credit Derivatives: Overview and Hedge-Based Pricing	7
2.1 The emergence of a new class of derivatives	7
2.2 Terminology	7
2.3 Underlying assets	10
2.3.1 Loans	10
2.3.2 Bonds	11
2.3.3 Convertible bonds	12
2.3.4 Counterparty risk	12
2.4 Asset swaps	12
2.5 Total return swaps	13
2.6 Credit default swaps	15
2.7 Hedge-based pricing	19
2.7.1 Hedge instruments	20
2.7.2 Short positions in defaultable bonds	20
2.7.3 Asset swap packages	22
2.7.4 Total return swaps	25
2.7.5 Credit default swaps	27
2.8 Exotic credit derivatives	37
2.8.1 Default digital swaps	37
2.8.2 Exotic default payments in credit default swaps	38

Contents

2.8.3	Rating-triggered credit default swaps	39
2.8.4	Options on defaultable bonds	40
2.8.5	Credit spread options	41
2.9	Default correlation products and CDOs	43
2.9.1	First-to-default swaps and basket default swaps	43
2.9.2	First loss layers	44
2.9.3	Collateralised debt obligations	46
2.10	Credit-linked notes	49
2.11	Guide to the literature	50
3	Credit Spreads and Bond Price-Based Pricing	51
3.1	Credit spreads and implied default probabilities	52
3.1.1	Risk-neutral probabilities	52
3.1.2	Setup	52
3.1.3	The fundamental relationship	54
3.1.4	The implied survival probability	54
3.1.5	Conditional survival probabilities and implied hazard rates	56
3.1.6	Relation to forward spreads	58
3.2	Recovery modelling	60
3.3	Building blocks for credit derivatives pricing	61
3.4	Pricing with the building blocks	64
3.4.1	Defaultable fixed-coupon bond	64
3.4.2	Defaultable floater	65
3.4.3	Variants of coupon bonds	66
3.4.4	Credit default swaps	66
3.4.5	Forward start CDSs	68
3.4.6	Default digital swaps	68
3.4.7	Asset swap packages	69
3.5	Constructing and calibrating credit spread curves	69
3.5.1	Parametric forms for the spread curves	70
3.5.2	Semi-parametric and non-parametric calibration	72
3.5.3	Approximative and aggregate fits	74
3.5.4	Calibration example	75
3.6	Spread curves: issues in implementation	77
3.6.1	Which default-free interest rates should one use?	77
3.6.2	Recovery uncertainty	79
3.6.3	Bucket hedging	81
3.7	Spread curves: discussion	82
3.8	Guide to the literature	83
Mathematical Background		85
4.1	Stopping times	86
4.2	The hazard rate	87
4.3	Point processes	
4.4	The intensity	
4.5	Marked point processes and the jump measure	91

	Contents	vii
4.6 The compensator measure	93	
4.6.1 Random measures in discrete time	95	
4.7 Examples for compensator measures	97	
4.8 Ito's lemma for jump processes	100	
4.9 Applications of Ito's lemma	101	
4.9.1 Predictable compensators for jump processes	102	
4.9.2 Ito product rule and Ito quotient rule	103	
4.9.3 The stochastic exponential	104	
4.10 Martingale measure, fundamental pricing rule and incompleteness	105	
4.11 Change of numeraire and pricing measure	107	
4.11.1 The Radon-Nikodym theorem	107	
4.11.2 The Girsanov theorem	108	
4.12 The change of measure/change of numeraire technique	109	
5 Advanced Credit Spread Models	111	
5.1 Poisson processes	111	
5.1.1 A model for default arrival risk	111	
5.1.2 Intuitive construction of a Poisson process	112	
5.1.3 Properties of Poisson processes	113	
5.1.4 Spreads with Poisson processes	115	
5.2 Inhomogeneous Poisson processes	115	
5.2.1 Pricing the building blocks	117	
5.3 Stochastic credit spreads	118	
5.3.1 Cox processes	119	
5.3.2 Pricing the building blocks	125	
5.3.3 General point processes	126	
5.3.4 Compound Poisson processes	128	
6 Recovery Modelling	131	
6.1 Presentation of the different recovery models	132	
6.1.1 Zero recovery	132	
6.1.2 Recovery of treasury	133	
6.1.3 Multiple defaults and recovery of market value	135	
6.1.4 Recovery of par	141	
6.1.5 Stochastic recovery and recovery risk	143	
6.1.6 Common parametric distribution functions for recoveries	147	
6.1.7 Valuation of the delivery option in a CDS	148	
6.2 Comparing the recovery models	150	
6.2.1 Theoretical comparison of the recovery models	150	
6.2.2 Empirical analysis of recovery rates	159	
7 Implementation of Intensity-Based Models	165	
7.1 Tractable models of the spot intensity	166	
7.1.1 The two-factor Gaussian model	167	
7.1.2 The multifactor Gaussian model	171	
7.1.3 Implied survival probabilities	172	
7.1.4 Payoffs at default	174	

Contents

7.2	The multifactor CIR model	174
7.2.1	Bond prices	175
7.2.2	Affine combinations of independent non-central chi-squared distributed random variables	176
7.2.3	Factor distributions	178
7.3	Credit derivatives in the CIR model	179
7.3.1	Default digital payoffs	180
7.3.2	Calculations to the Gaussian model	180
7.3.3	Calculations to the CIR model	184
7.4	Tree models	187
7.4.1	The tree implementation: inputs	187
7.4.2	Default branching	188
7.4.3	The implementation steps	190
7.4.4	Building trees: the Hull-White algorithm	190
7.4.5	Fitting the tree: default-free interest rates	193
7.4.6	Combining the trees	194
7.4.7	Fitting the combined tree	197
7.4.8	Applying the tree	198
7.4.9	Extensions and conclusion	199
7.5	PDE-Based implementation	200
7.6	Modelling term structures of credit spreads	204
7.6.1	Intensity models in a Heath, Jarrow, Morton framework	206
7.7	Monte Carlo simulation	211
7.7.1	Pathwise simulation of diffusion processes	214
7.7.2	Simulation of recovery rates	219
7.8	Guide to the literature	220
8	Credit Rating Models	223
8.1	Introduction	223
8.1.1	Empirical observations	224
8.1.2	An example	225
8.2	The rating process and transition probabilities	226
8.2.1	Discrete-time Markov chains	229
8.2.2	Continuous-time Markov chains	229
8.2.3	Connection to Poisson processes	231
8.3	Estimation of transition intensities	233
8.3.1	The cohort method	233
8.3.2	The embedding problem: finding a generator matrix	234
8.4	Direct estimation of transition intensities	238
8.5	Pricing with deterministic generator matrix	239
8.5.1	Pricing zero-coupon bonds	239
8.5.2	Pricing derivatives on the credit rating	240
8.5.3	General payoffs	241
8.5.4	Rating trees	242
8.5.5	Downgrade triggers	243
8.5.6	Hedging rating transitions	245

8.6	The calibration of rating transition models	246
8.6.1	Deterministic intensity approaches	246
8.6.2	Incorporating rating momentum	249
8.6.3	Stochastic rating transition intensities	250
8.7	A general HJM framework	251
8.8	Conclusion	253
9	Firm Value and Share Price-Based Models	255
9.1	The approach	255
9.1.1	Modelling philosophy	255
9.1.2	An example	256
9.1.3	State variables and modelling	259
9.1.4	The time of default	261
9.2	Pricing equations	263
9.2.1	The firm's value model	263
9.2.2	The pricing equation	264
9.2.3	Some other securities	265
9.2.4	Hedging	268
9.3	Solutions to the pricing equation	269
9.3.1	The T -forward measure	269
9.3.2	Time change	270
9.3.3	The hitting probability	270
9.3.4	Putting it together	271
9.3.5	The Longstaff-Schwartz results	271
9.3.6	Strategic default	273
9.4	A practical implementation: KMV	275
9.4.1	The default point	275
9.4.2	The time horizon	275
9.4.3	The initial value of the firm's assets and its volatility	275
9.4.4	The distance to default	276
9.5	Unobservable firm's values and CreditGrades	277
9.5.1	A simple special case: delayed observation	280
9.5.2	The idea of Lardy and Finkelstein: CreditGrades and E2C	281
9.6	Advantages and disadvantages	284
9.6.1	Empirical evidence	284
9.6.2	Discussion	286
9.7	Guide to the literature	286
10	Models for Default Correlation	289
10.1	Default correlation basics	290
10.1.1	Empirical evidence	290
10.1.2	Terminology	291
10.1.3	Linear default correlation, conditional default probabilities, joint default probabilities	292
10.1.4	The size of the impact of default correlation	293
10.1.5	Price bounds for FtD swaps	293
10.1.6	The need for theoretical models of default correlations	297

10.2	Independent defaults	298
10.2.1	The binomial distribution function	298
10.2.2	Properties of the binomial distribution function	299
10.2.3	The other extreme: perfectly dependent defaults	300
10.3	The binomial expansion method	301
10.4	Factor models	305
10.4.1	One-factor dependence of defaults	305
10.4.2	A simplified firm's value model	305
10.4.3	The distribution of the defaults	307
10.4.4	The large portfolio approximation	309
10.4.5	Generalisations	312
10.4.6	Portfolios of two asset classes	313
10.4.7	Some remarks on implementation	314
10.5	Correlated defaults in intensity models	315
10.5.1	The intensity of the default counting process	315
10.5.2	Correlated intensities	316
10.5.3	Stress events in intensity models	318
10.5.4	Default contagion/infectious defaults	321
10.6	Correlated defaults in firm's value models	321
10.7	Copula functions and dependency concepts	326
10.7.1	Copula functions	327
10.7.2	Examples of copulae	330
10.7.3	Archimedean copulae	333
10.8	Default modelling with copula functions	337
10.8.1	Static copula models for default correlation	337
10.8.2	Large portfolio loss distributions for Archimedean copulae	340
10.8.3	A semi-dynamic copula model	343
10.8.4	Dynamic copula-dependent defaults	349
	Bibliography	361
	Index	369