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# Incomplete Information and Heterogeneous Beliefs in Continuous-time Finance

With 43 Figures  
and 8 Tables



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# Table of Contents

<b>Incomplete Information: An Overview</b>	. . . . .	1	
1.1	Introduction	. . . . .	1
1.2	Portfolio Choice	. . . . .	1
1.2.1	Gennotte's Model	. . . . .	2
1.2.2	The Inference Process	. . . . .	4
1.2.3	Optimal Portfolio Choice	. . . . .	4
1.2.4	An Example	. . . . .	7
1.2.5	The Short Interest Rate	. . . . .	9
1.3	The Term Structure of Interest Rates	. . . . .	10
1.3.1	Dothan and Feldman's Models	. . . . .	10
1.3.2	A Characterization of the Term Structure	. . . . .	11
1.4	Equilibrium Asset Pricing	. . . . .	15
1.4.1	Honda's Model	. . . . .	16
1.4.2	The Equilibrium Price Process	. . . . .	17
1.5	Conclusion and Outlook	. . . . .	19
<b>The Impact of Incomplete Information on Utility, Prices, and Interest Rates</b>	. . . . .	23	
2.1	Introduction	. . . . .	23
2.2	The Model	. . . . .	25
2.3	Equilibrium	. . . . .	27
2.3.1	The Equilibrium Expected Lifetime Utility	. . . . .	27
2.3.2	The Equilibrium Price	. . . . .	28
2.3.3	The Equilibrium Interest Rate	. . . . .	30
2.4	Logarithmic Utility	. . . . .	31
2.4.1	The Equilibrium Expected Lifetime Utility	. . . . .	31
2.4.2	The Equilibrium Price	. . . . .	32
2.4.3	The Equilibrium Interest Rate	. . . . .	33
2.5	Power Utility	. . . . .	35
2.5.1	The Equilibrium Expected Lifetime Utility	. . . . .	35
2.5.2	The Equilibrium Price	. . . . .	40
2.5.3	The Equilibrium Interest Rate	. . . . .	48
2.5.4	Hedging Demand and the Equilibrium Price of Estimation Risk	. . . . .	50
2.6	Information, Utility, Prices, and Interest Rates: A Synthesis	. . . . .	51

## XII Table of Contents

2.6.1	Expected Lifetime Utility . . . . .	51
2.6.2	Share Prices . . . . .	52
2.6.3	Interest Rates . . . . .	53
2.7	Time-Varying Parameters . . . . .	55
2.7.1	The Equilibrium Expected Lifetime Utility . . . . .	56
2.7.2	The Equilibrium Price . . . . .	58
2.7.3	The Equilibrium Interest Rate . . . . .	58
2.8	Conclusion . . . . .	61
<b>3</b>	<b>Optimal Portfolio Choice Under Heterogeneous Beliefs</b> . . . . .	65
3.1	Introduction . . . . .	65
3.2	The Model . . . . .	67
3.3	The Deviant Agent's Problem . . . . .	70
3.4	Optimal Portfolio Choice . . . . .	71
3.5	An Example . . . . .	74
3.6	Conclusion . . . . .	78
<b>4</b>	<b>Optimal Consumption Under Heterogeneous Beliefs</b> . . . . .	81
4.1	Introduction . . . . .	81
4.2	The Cox-Huang Methodology . . . . .	82
4.3	Heterogeneous Beliefs . . . . .	84
4.4	An Example . . . . .	86
4.4.1	The Model . . . . .	86
4.4.2	Optimal Consumption Patterns Under Heterogeneous Beliefs . . . . .	87
4.4.3	An Algebraic Solution . . . . .	95
4.4.4	The Effect of the Time Horizon . . . . .	102
4.5	Portfolios and Consumption: A Synthesis . . . . .	105
4.6	Conclusion . . . . .	107
<b>5</b>	<b>Equilibrium Asset Pricing Under Heterogeneous Beliefs</b> . . . . .	109
5.1	Introduction . . . . .	109
5.2	The Model . . . . .	111
5.3	Equilibrium Consumption . . . . .	113
5.4	Equilibrium Prices . . . . .	116
5.4.1	The Equilibrium State-Price Density . . . . .	117
5.4.2	The Equilibrium Short Rate . . . . .	123
5.4.3	The Equilibrium Yield Curve . . . . .	128
5.4.4	The Equilibrium Share Price . . . . .	133
5.4.5	Equilibrium Option Prices and the "Smile Effect" . . . . .	139
5.5	Implied Risk Aversion . . . . .	145
5.6	Conclusion . . . . .	146

<b>6 Costly Information, Imperfect Learning, and Information Aggregation . . . . .</b>	149
6.1 Introduction . . . . .	149
6.2 The Model . . . . .	151
6.2.1 The Economy . . . . .	151
6.2.2 The Inference Process: Imperfect Learning . . . . .	152
6.3 Portfolio Choice under Costly Information . . . . .	154
6.3.1 The Agent's Problem . . . . .	154
6.3.2 The Agent's Optimal Investment and Research Policy .	155
6.3.3 Determinants of the Demand for Information . . . . .	157
6.3.4 Diversification and Information Costs . . . . .	160
6.4 Equilibrium Asset Pricing . . . . .	161
6.5 Information Aggregation and the Equity Premium . . . . .	165
6.6 Conclusion . . . . .	170
<b>7 Summary and Conclusion . . . . .</b>	173
<b>A Conditional Mean and Variance of <math>\ln(jEs)</math> . . . . .</b>	179
<b>B Conditional Mean and Variance of <math> n(x_8)</math> with Time-Varying Parameters . . . . .</b>	181
<b>C The Short Rate Under Heterogeneous Beliefs . . . . .</b>	183
<b>References . . . . .</b>	187
<b>List of Figures . . . . .</b>	191
<b>List of Tables . . . . .</b>	193
<b>List of Symbols . . . . .</b>	195